



## Original Research Article

### A discrete mathematical model for water supply management system with special reference to Kerala, India

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#### A B S T R A C T

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As water has become an essential commodity, even though the vessels of the nature are almost replete with it, the alacrity in providing potable water is emerging as an exigent need of the time. There are enormous parameters which are related with each other and having direct relationship with the environment. As India being the emerging country in the third world scenario, development of a comprehensive action plan to balance the water demand with the supply would help its sustainable development. In the present water supply management system anomalies and discrepancies can be visualized in plethora. The impact of this is having intensive reflections in the region where the consumption is more with respect to the population density. The study area is the southern state of India, Kerala, which is being called the *Gods own Country* in consideration of its natural beauty and the potential of natural resources including the water resources. The state is blessed with 44 rivers as nature's gift but the efficiency in providing water in the potable form is less than 40%. In an adept perspective if we analyze the problem it can be discerned that the lacuna fraught with the pragmatic procedures are mainly because of the negligence of considering the various variables which are having direct and indirect impacts on the system. Hence this attempt is to ferret out the associated variables and its cross impact relationship with respect to the timely changes.

## Introduction

The consumption of water by an individual is required for various physiological process such as blood formation, food assimilation etc. The quantity of water that an individual of the present scenario in Kerala in average of 2 liters per day. Even though it is a very small quantity it is not essential to supply water for drinking purpose with high degree of purity. In a water supply system there are various variables those having influence in

the rate of demand of water. These factors have to be carefully and properly analyzed before arriving the rate of demand. The existing water supply system is not meeting with the requirements properly because of the drastic changes happens in the environmental conditions as well as the population dynamics. The relationship between the water supply and water demand is depending upon enormous direct and

indirect variables which are always exhibiting non linear changes with respect to the time factor. While taking in the case of the state of Kerala where the population density is changing rapidly along with the living standards. Here the functional difference between the rural and urban life pattern is very merger so that the water consumption pattern is almost similar which require equal attention. Another important feature for the water consumption is that the rainfall distribution along with the land use pattern. Apart from the drinking requirement more water is used for the irrigation and lifestyle management. The system of sanitation operating at the of Kerala is having unique features such that apart from the other states of India the cultural ethics imparts more influence on the conventional sanitation procedure along with the personal hygiene. Hence the water supply management procedures should consider all these aspects while formulating a comprehensive system to meet the requirements. On analyzing these situations the experts working in the particular realm The inter relationship between these parameters and the analysed and the scenario can be made using the tool based on Discrete Mathematical Modeling. The significance of these parameters can be analyzed using the exert opinion and cross impact analysis which helps to formulate a comprehensive model used for representing the system.

### **Study area**

The Kerala state has an area of 38,863 km<sup>2</sup> and is bordered by Karnataka to the north and northeast, Tamil Nadu to the south and southeast and the Arabian Sea towards the west. Rivers are the main source of supply of drinking water in the State. There are 44 rivers in the State. All these rivers cater to the drinking water requirements in the State.

There are a number of abstraction points in all the major rivers, which are used for supply of drinking water and industrial water supply. Karamana River is the main source of water supply to the Thiruvananthapuram City, the Nedumangad municipality and the adjoining panchayaths. The intakes for many water supply schemes including 6 rural water supply schemes exists in the upstream reach of the river. There are a number of water supply schemes including 10 rural water supply schemes in the upstream and midstream of Kallada River maintained by Kerala Water Authority. These are used for supplying water to Kundara and adjoining panchayaths. The upper stretches of Achenkovil River and its tributaries above Mavelikkara are used for rural water supply schemes maintained by Kerala Water Authority. The Pamba River, upstream of Chengannur, is used for many rural water supply schemes without conventional treatment but only disinfection and the downstream of Chengannur up to confluence with Vembanad backwaters, is used for drinking after conventional treatment. The upstream and downstream of Manimala river stretch is used for drinking without conventional treatment but after disinfection and drinking after conventional treatment after disinfection respectively.

There are 13 water supply schemes including 9 rural water supply schemes operate from Meenachil River. There are 15 water supply schemes in Muvattupuzha River out of which 8 are rural schemes. The Periyar River, upstream above Kalady and downstream of Kalady to Pathalam bund is used for drinking purpose without conventional treatment after disinfection and after conventional treatment after disinfection respectively. A number of water supply schemes exist in the upstream of Chalakudy River. The Bharathapuzha River,

upstream part above Kuttippuram and downstream of Kuttippuram upto Chamravattom ferry, is used for water supply schemes. 20 water supply schemes exist in Chaliyar River and 15 are under construction or proposed. The upstream river stretch of Valapattanam River is also used for drinking purpose under water supply schemes. 70 percent of houses in Kerala lack access to drinking water in the state. The problem in Kerala is not a one-sided one, it is rather multifaceted.

## Materials and Methods

### Discrete Mathematical Model

We are using the discrete mathematical model digraphs such as weighted and signed digraphs for diagrammatic representation of the interrelationship between the identified variables. With the help of the pulse process it can be the possible changes that will be encountered by the system with the changes in the time factor and fluctuations in each pulse can be analyzed systematically. The signed digraph model has its many simplifications. For eg, some effects of variables on others are stronger than other effects. The signed digraph model assumes that all effects are equally strong, by placing weights of equal units magnitude on each arc  $(u,v)$ , thus yielding weighted digraph. There are many constraints happened to be introduced while engaging forecasting technique and measuring the stability of the system for a particular time interval. From these constraints there is a need to find out alternative strategies which will allow to meet the constraints. If the system is a weighted digraph some of the possible changes or strategies are following.

1. Change the value of certain vertices at the specified times

2. Add at given time a new vertex (Institution) and new arcs to and from it (relations of interactions of the institutions with existing ones)
3. Change the sign of a given arc at a given time
4. Change the weight of a given arc at a given time
5. Add a new arc between existing vertices
6. Delete arc between existing vertices
7. Add a new cycle (deviation amplifying or deviation counteracting)

These methods are generally used for making strategies for meeting the constraints.

### Pulse process

To make somewhat deeper analysis of the weighted digraph model it is necessary to make some very specific assumptions about the effect that changes in value in one vertex have other vertices. It shall be called such assumptions change of value rules. The specific change of value rules assumed plays a rather subtle role in its relations to our conclusions. If we assume that the basic data (say for example initial values at each vertex and weights) are known only imprecisely, then the ultimate predictions based on specific change of values rule will be imprecise as well. Any conclusions drawn should be regarded as tentative and subjected to a sensitivity analysis. Such an analysis will involve redoing the modeling with changes in the basic data and perhaps using different changes of values. We shall present several theorems about stability of a weighted, signed digraph under pulse process. These theorems can be applied in testing for the stability of a digraph  $D$  reduces to asking simple questions about the Eigen-values of  $D$ . The first theorem says that we simply have to calculate the

magnitudes of the Eigen- values in order to draw some interesting conclusions.

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## **Results and Discussion**

In this work a humble attempt has made to describe the nature and the behavior of the water supply management in the developing tropical urban net. A mathematical is formulated for the system and with the help of this we can contemplate the dynamics of the system. As the water management is a macro frame to study the interrelationship between the variables. According to Eckstern . O 1958) It is recommended that there should be a set of parameters which are having direct and indirect relationship with in the dynamics of the water supply system. Maxwell M.H made an attempt to list out variables which impart direct impact on any type of water supply management system.

The model that we have developed is a discrete mathematical model the signed and weighted digraph model and the stability of the system is analyzed under the pulse process. We have made the following observations regarding this model.

1. Both the weighted and signed digraph model for the water supply management system showed a high degree of complexity due to the presence of a number of factors which interacts through the feedback cycles
2. The signed model is pulse and value unstable and failed to find out the stabilizing factors which can be adopted in meaningful way, as far as the national policy is concerned.
3. The original weighted digraph model is both pulse and value unstable but we can perform meaningful stabilizing strategies.

On the 14 system elements, the Industrial Development (ID) and Living Standard (LS) are acting through maximum number of feedback cycles. The change affecting these vertices does not affect the stability since the number of positive and negative balance each other.

1. Looking at the important relationship affecting the stability we observed that Population (Pp) and Water Resources (WR), System of Supply (SS) and Water Resources (WR), Technology Advancement (TA) and System of Sanitation (Sa) , System of Supply (SS) and Water Pressure(WP), etc .are significant. Changing the interrelationship between the variables from the conventional pattern may mitigate or efface characteristics changes in the water supply management systems.

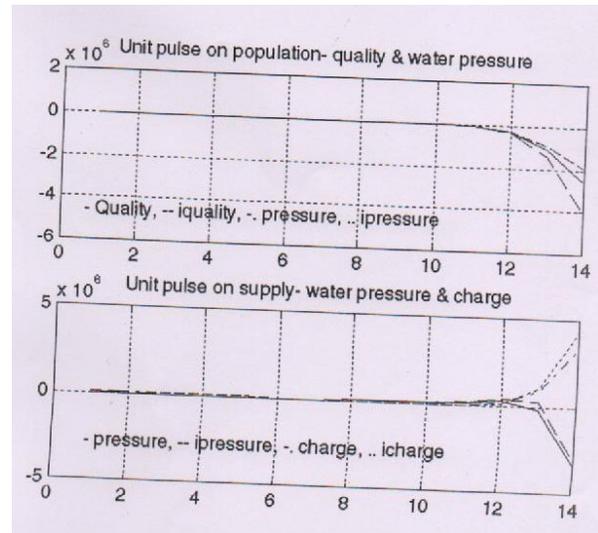
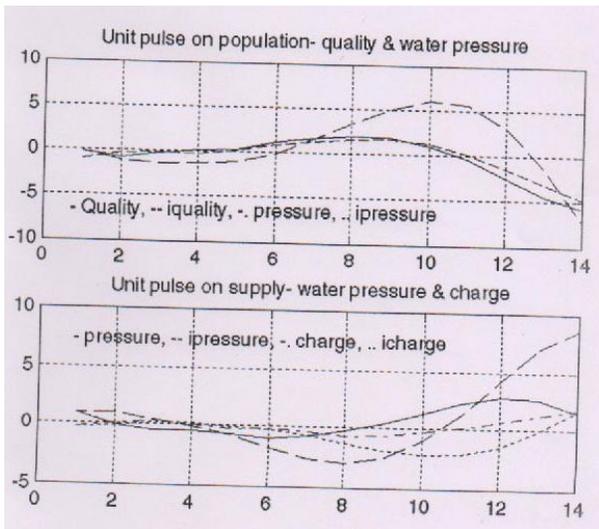
2. The cumulative effects of
  - a. The Population (Pp) with Water Resources (WR), Quality of Water (WQ), Water Pressure in the Distribution Pipe (WP)
  - b. Apply with the Water Resources (WR), Water Pressure in the Distribution Pipe (WP) and the Water Charge (WC)
  - c. Urbanization(Ur) with System of Sanitation (Sa) and Water Charge (WC) Water Resources (WR) with Urbanization(Ur), Water Pressure in the Distribution Pipe (WP)
  - d. Technology Advancement (TA) with Urbanization (Ur), System of Sanitation (Sa), Water Charge (WC)
  - e. System of Sanitation (Sa) with Water Resources (WR) and Water Charge (WC)
  - f. Demand (Dm) with Water Resources (WR) and Quality of Water (WQ) and Water Charge (WC)
3. Apart from other analysis conducted by measuring the influence of these variables among the other variables, as we have seen that the population with water resources, technology with the system sanitation supply with water pressure in the distribution pipe demand and water resources, demand with quality of water and the demand with the charge are the variables that should be concentrated on policy formulations.
1. The emerging policy implications are water conservation must be given utmost important in high density state like Kerala. Activities like land reclamation should be badly affect the water resources must be discouraged
2. Besides the conservation of water resources, water quality management also must be given utmost importance
3. For the system of sanitation optimization strategies should be one of the major focal point of the policy formulations the technological advancements acts as catalyzing agent for sanitation activities. Hence standard norms should be providing along with the awareness campaign for the system of sanitation regarding its operations and management.
4. Water charge is also important regulatory factor in water supply management system. Hence a pragmatic approach should be incorporated with the assessment of water charge, hence any change in the conventional pattern of water pricing should be observed in accordance with the developing context.
5. With urbanization and increased water demand the pressure heads in the distribution pipes decreases significantly at peak time while it is high during the other times. This necessitates adoption of better technology for instance using high quality pipes of sufficient diameter. The augmentation scheme should be favored to balance the supply and water pressure for that a detailed assessment should be conducted in the field of supply network to identify the areas where the water pressure and the supply are demonstrating lack of alacrity.

**Table 1.** The major variables which play a vital role in the dynamics of the Water Supply System

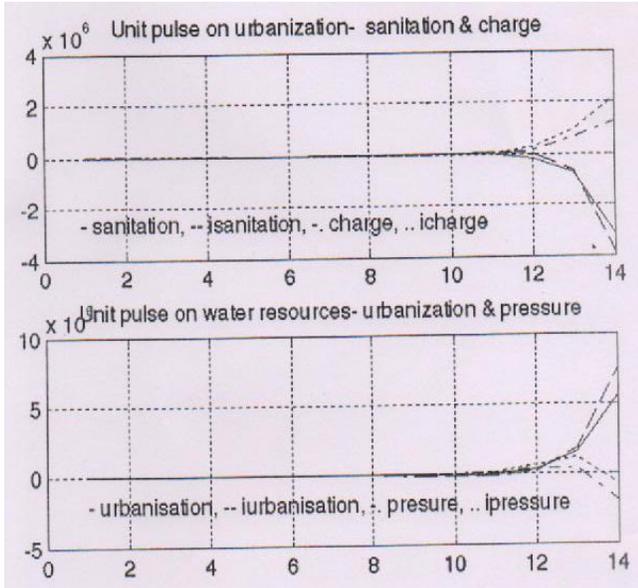
SI No	Variables	Code of Representation
1	Population	Pp
2	Industrial Development	ID
3	Living Standard	LS
4	Rainfall	Rf
5	Urbanization	Ur
6	System of Supply	SS
7	Water Resources	WR
8	Quality of Water	WQ
9	System of Sanitation	Sa
10	Water Charge	WC
11	Water Pressure in the Distribution Pipes	WP
12	Technological Advancement	TA
13	Demand	Dm
14	Land Use	LU

Digraph 1  
Unit pulse on population, Water Quality

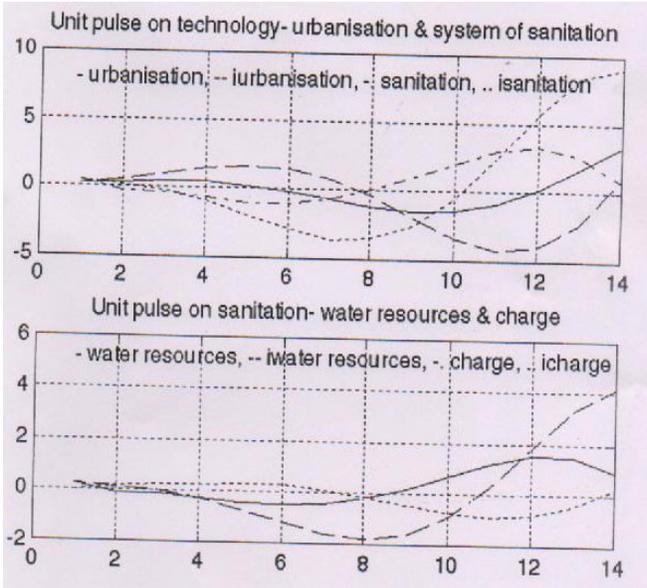
Digraph 2  
Unit pulse on Urbanization, Sanitation & Water Charge & Water



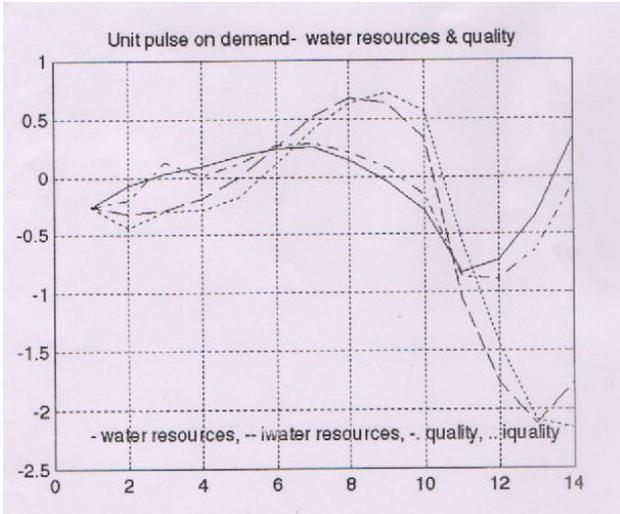
Digraph 3  
Unit pulse on Urbanization, Sanitation Water Resources & Water



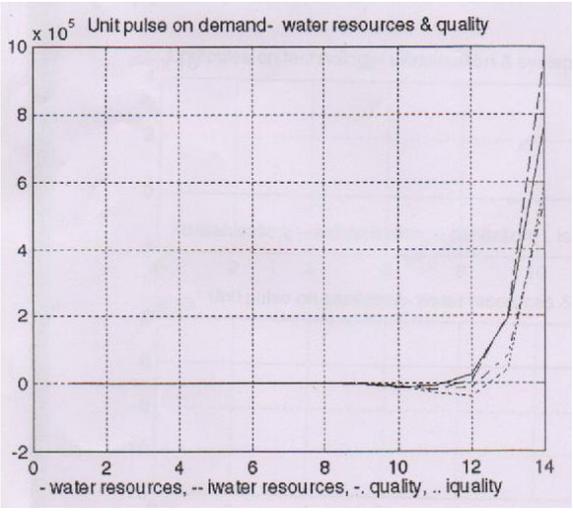
Digraph 4  
Unit pulse on Technology, Urbanization Water Resources &



Digraph 5  
Unit pulse on Demand, Water Resources & Water Quality



Digraph 6  
Unit pulse on Demand, Water Resources & Water Quality with



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